readily obtained. In numerical problems the task of solving the transcendental equations (7) or (10), and (7a) presents no practical difficulty.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

172. Proposed by H. C. FEEMSTER, York, Neb.

Show that $\frac{(nr)!}{n!(r!)^n}$ is an integer.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

By induction we get the following:
Suppose $r(n-1)!$ is divisible by $(r!)^{n-1}(n-1)!$. Now

$$\frac{(rn)!}{n!(r!)^n} \div \frac{r(n-1)!}{(r!)^{n-1}(n-1!} = \frac{(nr)!}{r\ (n-1)!} \times \frac{(n-1)\ !}{n!r!}$$

$$= \frac{nr(nr-1)(nr-2)\ldots\text{to } r \text{ factors}}{nr(r-1)!}$$

$$= \frac{(nr-1)(nr-2)\ldots\text{to } (r-1) \text{ factors}}{(r-1)!} = \text{an integer.}$$

Now $\frac{r!}{r!\ 1!}$=an integer, and hence $\frac{(2r)!}{(r!)^2 2!}$=an integer; $\frac{(3r)!}{(r!)^3 3!}$=an integer, and so on.

173. Proposed by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Find integral values satisfying the equation,

$$a_1^2+a_2^2+a_3^2+\ldots+a_n^2=d^4.$$

I. Solution by FRANK LOXLEY GRIFFIN, Ph. D., Williams College.

One set of solutions may be obtained by putting $a_1=df_1$, $a_2=df_2$, etc., which reduces the problem to that of finding a sequence of n integers, the sum of whose squares is a perfect square. Or, geometrically, we seek $n-1$ right triangles whose sides are all integers, and the hypotenuse of each being one leg of the next.

I. This is readily accomplished for $n=2$ by recalling that, if (p^2-q^2) and $2pq$ are legs, where p and q are integers, the hypotenuse is also an integer, (p^2+q^2). Let f_1 be *any odd integer* (>1) and take $p-q=1$ and $p+q=f_1$, so that $p=\frac{1}{2}(f_1+1)$, $q=\frac{1}{2}(f_1-1)$, both integers. Thus the sides

of the right triangle are f_1, $\frac{1}{2}(f_1^2-1)$, and $\frac{1}{2}(f_1^2+1)$. Or, calling the second f_2 and the third d, we have $d^2=f_1^2+f_2^2$.

II. Let this value of d for $n=2$ be denoted by d_2, and observe that d_2 is *odd*. For f_1=say, $2m+1$ (being odd), whence $d_2=2m^2+2m+1$, odd. Thus d_2 may be treated precisely as was f_1, giving $f_3=\frac{1}{2}(d_2^2-1)$, $d_3=\frac{1}{2}(d_2^2+1)$, or $d_3^2=d_2^2+f_3^2=f_1^2+f_2^2+f_3^2$.

Since again d_3 is odd, the process may clearly be continued indefinitely. The successive values of f and d may be found from: $2f_{n+1}=d_n^2-1$, $2d_{n+1}=d_n^2+1$. The following table gives the values up to $n=6$ if we begin with f_1+3:

f_n	3	4	12	84	3612	6526884
d_n		5	13	85	3613	6526885

To obtain the proposed a's for any value of n, merely multiply each f (up to f_n) by d_n. Thus, for $n=3$,

$$39^2+52^2+156^2=13^4.$$

[We might also start with any *even* integer (>2), and either (A) form a sequence all terms of which are even multiples of the terms obtaining by beginning with any odd factor of f_1, or (B) let $f_1=2pq$ and take for p, q *any* factors whose product is $\frac{1}{2}f_1$. Thus for $f_1=14$:

$$p=7,\quad q=1,\quad f_2=48,\quad d_2=50,$$
$$p_2=25,\quad q_2=1,\quad f_3=624,\quad d_3=626, \text{ etc.}]$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Dr. Martin's solution can be extended to suit this requirement.

Let $x_1^2+x_2^2+x_3^2+\ldots+x_n^2=c^2$.

Then $c^2-n^2=(c-n)(c+n)=x_1^2+x_2^2+x_3^2+\ldots+x_{n-1}^2$.

Let $c-n=b$, then $c+n=(x_1^2+x_2^2+x_3^2+\ldots+x_{n-1}^2)/b$.

$\therefore x_n=\frac{1}{2}(x_1^2+x_2^2+x_3^2+\ldots+x_{n-1}^2-b^2)/b$.

$c=\frac{1}{2}(x_1^2+x_2^2+x_3^2+\ldots+x_{n-1}^2+b^2)/b$.

Multiplying through by c^2 after substituting, we get

$$(cx_1)^2+(cx_2)^2+(cx_3)^2+\ldots+[(c/2b)(x_1^2+x_2^2+x_3^2+\ldots+x_{n-1}^2-b^2)]^2$$
$$=[(1/2b)(x_1^2+x_2^2+x_3^2+\ldots+x_{n-1}^2+b^2]^4.$$

$$\therefore (4b^2cx_1)^2+(4b^2cx_2)^2+(4b^2cx_3)^2+\ldots[2bc(x_1^2+x_2^2+x_3^2+\ldots$$
$$+x_{n-1}^2-b^2)]^2=(x_1^2+x_2^2+x_3^2+\ldots+x_{n-1}^2+b^2)^4.$$

Let $n=7$, $b=3$, x_1, x_2, x_3, ...=1, 2, 3, ..., 6. Then $c=\frac{50}{3}$.

$\therefore (600)^2+(1200)^2+(1800)^2+(2400)^2+(3000)^2+(3600)^2+(8200)^2=(100)^4$, or $6^2+12^2+18^2+24^2+30^2+36^2+82^2=10^4$.

Also solved by the Proposer.